kth-order sum-free functions and nonvanishing k-flats

A *kth-order sum-free function* is a function $F\colon \mathbb{F}_2^n \to \mathbb{F}_2^m$ such that

$$\sum_{x\in A} F(x) \neq 0$$

for every k-dimensional affine subspace (k-flats) A of $\mathbb{F}_2^n$. These functions were recently introduced by Carlet (2025) as a natural generalization of almost perfect nonlinear (APN) functions. APN functions are precisely the 2nd-order sum-free and have been studied extensively since their introduction in 1993. For APN function, we always have $m = n$ but for $k > 2$, no results on the minimum possible value of m are currently known.

For an arbitrary function F, we call a k-flat A *nonvanishing with respect to F* if it satisfies the above condition *nonvanishing* . So far, only nonvanishing 2-flats have been investigated, for example by Li et al. (2020).

In this talk, I will present a new approach to studying the nonvanishing k-flats of a function for arbitrary k. This approach shows that most kth-order sum-free functions give rise to an $(n-k)$th-order sum-free function. In addition, I will present the first lower bound on m such that a kth-order sum-free functions can exist. Finally, I establish a connection between kth-order sum-free functions and subcodes of the Reed-Muller code that avoid all minimum weight codewords. The latter two results are joint work with Philipp Heering and Vlad Taranchuk.

References

[1] P. Heering, C. Kaspers, and V. Taranchuk. *On Reed-Muller subcodes, Grassmannian partitions and sum-free functions.* 2026. arXiv: `2605.22958 [cs.IT]`.

[2] C. Kaspers. *Nonvanishing k-flats of Boolean and vectorial functions.* 2026. arXiv: `2603.28266 [math.CO]`.