Toroidal groups, generalized Jacobians and non-totally real number fields

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Abstract

A *toroidal group* is a complex Lie group on which every holomorphic function is constant.

In the first part of this talk, the relationship between toroidal groups and non-totally real number fields is introduced: under some necessary assumptions, any toroidal group $\mathcal{T}$ of complex dimension 2 and real rank 3 with extra multiplication is isogenous to the group $\mathbb{C}^2/\mu_\Phi(\mathcal{O}_K)$, where K is the non-totally real cubic number field $\mathrm{End}_0(\mathcal{T}) = \mathrm{End}(\mathcal{T}) \otimes_{\mathbb{Z}} \mathbb{Q}$ and μ_Φ is a Minkowski embedding. This correspondence is extended to higher dimension, writing down the relations between the periods defining a toroidal group $\mathcal{T}$ (of complex dimension n and real rank $n+1$) and the minimal polynomial of a primitive element of K. Furthermore, for such a toroidal group $\mathcal{T}$, I explicitly show the analytic and rational representations of its ring of endomorphisms. Lastly, I show some results concerning the relationship between a toroidal groups of complex dimennsion 3 and real rank 5 and a quintic field with two pair of complex embeddings.

In the second part of this talk, I introduce the (birationally) isomorphism between a toroidal group and the generalized Jacobian of an elliptic (or hyperlliptic) curve. Finally, I give an explicit description of the m-torsion in the geometric correspondence of a toroidal group $\mathcal{T} = \mathbb{C}^2/\mu_\Phi(\mathcal{O}_K)$ with a generalized Jacobian $\mathfrak{J}$ of an elliptic curve.

Keywords: Toroidal groups; Jacobians; number fields.
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