On a Group Action of Subgroups of PGL_2 on Monic Polynomials

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Abstract

Let K be a field, $K(x)$ the rational function field over K and $\mathrm{PGL}_2(K)$ the projective general linear group over K. We write $[A]$ for the coset of $A \in \mathrm{GL}_2(K)$ in $\mathrm{PGL}_2(K)$. For

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

we define $[A] \circ x = \frac{ax+b}{cx+d}$ and set

$$f^{[A]}(x) := \lambda_{[A],f}(cx+d)^{\deg(f)} f\left([A] \circ x\right),$$

where $\lambda_{[A],f} \in K^*$ makes the output-polynomial monic. It can be shown that this induces a right group action of $G \leq \mathrm{PGL}_2(K)$ on a subset of monic polynomials over K.

We present some recent developments about polynomials that are invariant under this action and show how they are related to *rational transformations* of polynomials, a concept arising in the context of the construction of irreducible polynomials over finite fields.

Keywords: Group Action, Factorization of Polynomials, Galois Theory