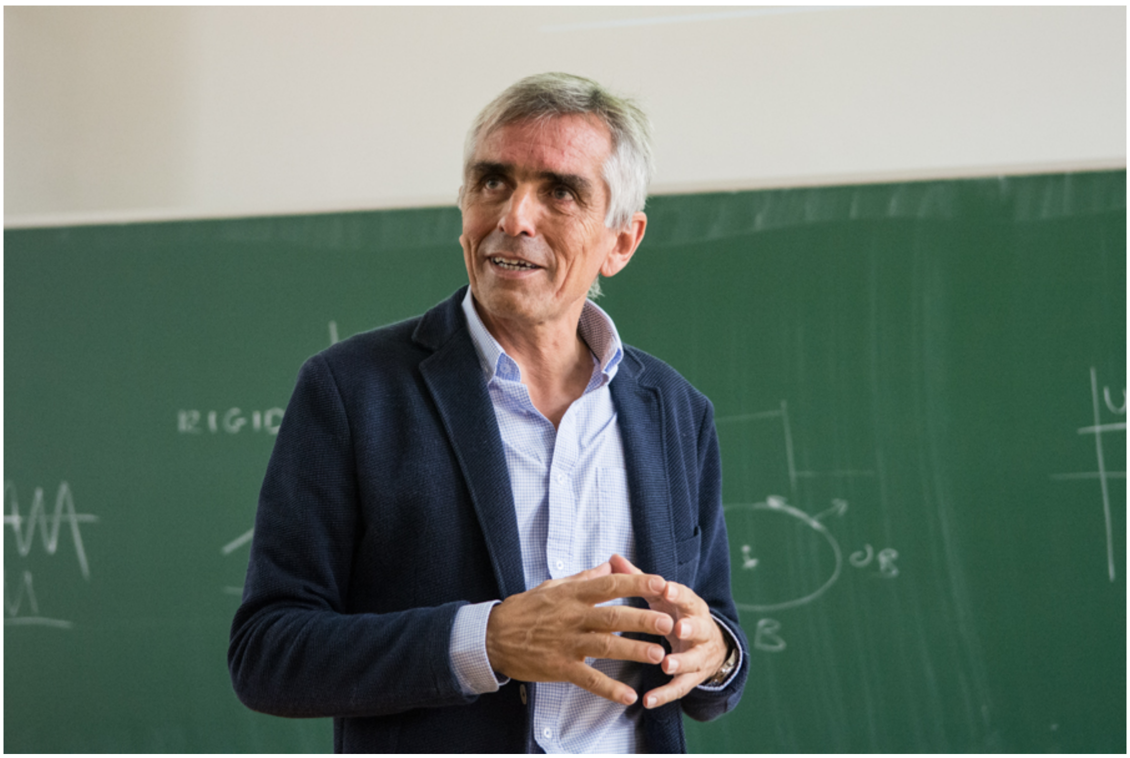
Book of Abstracts

Conference
Prague Compressible Meeting

In honor of the 60^{th} birthday of Eduard Feireisl



Institute of Mathematics, Czech Academy of Sciences
Prague, December 18-20, 2017

The event is supported by the European Research Council under project MATHEF 320078.

Convergence to travelling waves in Fisher's population genetics model with degenerate diffusion and a non-Lipschitzian reaction term

PETER TAKÁČ, P. DRÁBEK

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Abstract

We begin by a brief presentation of a well-known mathematical model for population genetics due to R. A. Fisher (1937) where a population is divided into three classes of genotypes, aa, AA, and aA. The mathematical model is represented by a reaction-diffusion equation for the unknown *relative density* $u(x,t)$ of the population of allele A at the point $x \in \mathbb{R}$ of the habitat at time $t \in \mathbb{R}_+$. An important question from Population Biology is if *genetic diversity* is preserved at certain location and at certain time. We will show that this is a "dynamic problem" that requires a dynamical system approach by convergence to a travelling wave.

Our first mathematical result will be on travelling waves with a degenerate or singular diffusion (like the p-Laplacian) and possibly nonsmooth (e.g., non-Lipschitzian), bi-stable reaction term. We will show that, in spite of nonuniqueness for the one-dimensional ordinary differential equation, the travelling wave $u(x,t) = v(x-ct)$ and its speed c are unique (up to a spatial shift in the argument). In order to answer our genetic diversity problem, we have to know if the two extreme values 0 and 1 of the solution (i.e., the profile of travelling wave) $v(x)$ are reached within a bounded spatial interval $[z_0, z_1] \subset \mathbb{R}$. Namely, the genetic diversity is lost in $(-\infty, z_0]$ thanks to $v = 0$, and in $[z_1, \infty)$ thanks to $v = 1$.

Our second result establishes the long-time convergence to a travelling wave for the classical Brownian diffusion (i.e., the linear Laplace operator) combined with a non-Lipschitzian reaction term. We will briefly explain the "mechanism" that yields the desired convergence.

Dear Friends,
welcome to the conference in celebration of Eduard Feireisl's 60^{th} birthday, and in honor of his contributions to compressible fluid mechanics.

Inside this booklet, you will find the schedule of lectures together with abstracts. The official program takes place in the **Blue lecture room** at the Institute of Mathematics (Žitná 25, Prague) and reads as follows:

- Monday, December 18

  8:15 - 8:45 Registration

  8:50 - 9:00 Opening by Jiří Rákosník (the director of the Institute of Mathematics)

  9:00 - 12:00 Lectures

  12:00 - 14:00 Lunch

  14:00 - 16:30 Lectures

- Tuesday, December 19

  9:00 - 12:00 Lectures

  12:00 - 14:00 Lunch

  14:00 - 15:30 Lectures

  16:00 - 16:30 Bernard Bolzano medal for Eduard Feireisl awarded by the President of the Czech Academy of Sciences

  16:30 - 22:00 Banquet

- Wednesday, December 20

  9:00 - 12:00 Lecture session

  12:00 - 14:00 Lunch

  14:00 - 15:30 Lectures

For your convenience, we have arranged lunch to be served every day at the Institute of Mathematics.

We hope you will enjoy the conference and wish you pleasant days full of mathematics and Christmas Prague!

Pavel Krejčí, Martin Michálek, Šárka Nečasová and Jiří Rákosník
the organizers of the conference

Contact: michalek@math.cas.cz, *mobile: +420 602 169 058*

Morning schedule

	Monday, December 18	Tuesday, December 19	Wednesday, December 20
8:50–9:00	Opening		
9:00–9:30	Antonín Novotný *Existence of weak solutions to compressible Navier-Stokes equations with large non-homogenous data*	Elisabetta Chiodaroli *The isentropic Euler equations: Riemann problems and non-uniqueness*	Piotr Gwiazda *Energy conservation for some compressible fluid models*
9:30–10:00	Marta Lewicka *Visualization of the convex integration solutions to the Monge-Ampère equation*	Emil Wiedemann *Localised Relative Energy*	Agnieszka Świerczewska-Gwiazda *Dissipative measure valued solutions for general hyperbolic conservation laws*
10:00–10:30	Ewelina Zatorska *Transport of congestion in two phase compressible/incompressible flow*	Jan Březina *New take on measure-valued solutions for complete Euler system*	Dorin Bucur *Optimal shapes with free discontinuities*
10:30–11:00	Coffee break	Coffee break	Coffee break
11:00–11:30	Peter Bella *Quantitative stochastic homogenization of elliptic systems*	Mária Lukáčová *The role of measure-valued solutions in compressible flows*	Dalibor Pražák *Regularity and uniqueness for a critical Ladyzhenskaya fluid*
11:30–12:00	Martina Hofmanová *On random distributions and applications to compressible fluids*	Christian Klingenberg *On non-uniqueness of the two dimensional compressible Euler equations*	Pavol Quittner *Entire solutions of a semilinear parabolic equation*
12:00–14:00	Lunch break	Lunch break	Lunch break

Afternoon schedule

	Monday, December 18	Tuesday, December 19	Wednesday, December 20
14:00–14:30	DONATELLA DONATELLI *On a low Mach number limit for supernovae*	JOSEF MÁLEK *On Euler/Navier-Stokes fluids*	YONGZHONG SUN *Global weak solution to 1D compressible MHD without resistivity*
14:30–15:00	GIULIO SCHIMPERNA *Some results on the functionalized Cahn-Hilliard equation*	PHILIPPE LAURENCOT *Self-similar solutions to a thin film Muskat problem*	PETER TAKÁČ *Convergence to travelling waves in Fisher's population genetics model with degenerate diffusion . . .*
15:00–15:30	ADRIEN PETROV *A rigorous derivation of the stationary compressible Reynolds equation via the Navier-Stokes equations*	ELISABETTA ROCCA *Diffuse interface models for multiphase tumor growth*	MILAN POKORNÝ *Compressible Navier-Stokes equations with entropy transport*
15:30–16:00	Coffee break		
16:00–16:30	PIERANGELO MARCATI *Splash singularity for a free-boundary incompressible viscoelastic fluid model*	Medal ceremony and banquet opening	

Quantitative stochastic homogenization of elliptic systems

Peter Bella

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Abstract

I will discuss homogenization of second-order uniformly elliptic equations (and systems) in divergence form with random coefficients

$$-\nabla \cdot a(\nabla u) = f,$$

where $a(x) = a(\omega, x)$ is a random coefficient field. Assuming the probability distribution on the space of coefficient fields is *stationary* (meaning for any $x \in \mathbb{R}^d$ the field a and $a(\cdot + x)$ have the same joint distribution) and *ergodic* (loosely speaking values of a near x and y are becoming independent as $|x-y| \to \infty$), a classical qualitative result [Papanicolaou & Varadhan '78, Kozlov '79] asserts that solutions u^ϵ to $-\nabla \cdot (a(x/\epsilon)\nabla u^\epsilon(x)) = f$ converge (as $\epsilon \to 0$) almost surely (i.e. for almost every coefficient field a) to the solution u_{hom} of a constant-coefficient (effective) equation. Assuming stronger *quantified* ergodicity on the probability distribution one can obtain a rate of convergence in ϵ. I will discuss several such results, including higher-order error estimates in weak norms as well as an analogue of multipole expansion of decaying solutions (Green's function) for the random setting. In the second half of the talk I will discuss large-scale regularity results for a-harmonic functions (solutions to the above equation with $f \equiv 0$), such as first-order Liouville principle and $C^{1,\alpha}$-estimate, and their extension to the case of elliptic and parabolic equations with degenerate and unbounded coefficients.

New take on measure-valued solutions for complete Euler system

Jan Březina

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Abstract

We consider the complete Euler system describing the time evolution of a general inviscid compressible fluid. In our previous work we introduced a concept of measure–valued solution based on the total energy balance and renormalization of entropy inequality for the physical entropy. Today we introduce a new concept of measure–valued solution without any renormalization and in conservative variables usual for numerical analysis. Both of these classes do satisfy the weak-strong uniqueness property. Furthermore the new class of so–called dissipative measure–valued solutions is large enough to include the vanishing dissipation limits of the Navier–Stokes–Fourier system. Our main result states that any sequence of weak solutions to the Navier–Stokes–Fourier system with vanishing viscosity and heat conductivity coefficients generates a dissipative measure-valued solution of the Euler system under some physically grounded constitutive relations. Finally, we discuss the same asymptotic limit for the bi-velocity fluid model introduced by H.Brenner.

Optimal shapes with free discontinuities

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Abstract

The question of finding the optimal shape of an obstacle subject to a volume constraint which minimizes the drag is a standard one. While in the literature it is classical to raise such problems in the context of the Stokes or Navier-Stokes equations and no-slip conditions on the boundary of the obstacle, we address a similar question provided Navier slip conditions occur at the boundary. The difference between the two problems is not marginal. While Dirichlet boundary conditions lead to a rather classical shape optimization problem with free boundary, the second one leads to a non-standard free discontinuity problem of Robin type. The shape of the obstacle is completely free and no a priori geometric constraints are imposed. As a consequence, structures of dimension $N-1$ having no volume, similar to the so called "sharklets", can pop up naturally.

In this talk, I will introduce the question, set the functional framework in the context of special functions of bounded variation and bring some answers in the scalar case.

The isentropic Euler equations: Riemann problems and non-uniqueness

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Abstract

In this talk we discuss some applications of the method of convex integration to the compressible Euler system of gas dynamics in two space dimensions. This leads to the construction of infinitely many non-standard solutions even in the case of classical Riemann data. We also show some recent studies meant at visualizing numerically such non-standard solutions.

On a low Mach number limit for supernovae

Donatella Donatelli, E. Feireisl

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Abstract

Fluid dynamic equations are used to model various phenomena arising from physics, engineering, astrophysics.

In particular these type of equations are useful to model some phenomena taking place at the level of supernovae, where the modeling equations are given by the coupling of the compressible Navier Stokes equations with equations that take into account of the chemical reactions and heat effects.

One feature of these flows is that they take place under a low Mach number and high Reynolds number regime and so they are affected by the presence of high oscillating acoustic waves. In order to understand this type of dynamic one has to derive a model for low speed flows (low Mach number) in a hydrostatically balanced, radially stratified background that removes acoustic waves and allows for the development of finite amplitude temperature and density variation.

Here, we analyze a simplified model for supernovae and we identify the asymptotic limit in the regime of low Mach, low Froude and high Reynolds number. The system is driven by a long range gravitational potential. We show convergence to an anelastic system for ill-prepared initial data. The proof is based on frequency localized Strichartz estimates for the acoustic equation based on the recent work of Metcalfe and Tataru.

References

[1] D. Donatelli and E. Feireisl, An anelastic approximation arising in astrophysics, *Math. Ann.*, **369**, (2017), 1573–1597.

Energy conservation for some compressible fluid models

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Abstract

A common feature of systems of conservation laws of continuum physics is that they are endowed with natural companion laws which are in such case most often related to the second law of thermodynamics. This observation easily generalizes to any symmetrizable system of conservation laws. They are endowed with nontrivial companion conservation laws, which are immediately satisfied by classical solutions. Not surprisingly, weak solutions may fail to satisfy companion laws, which are then often relaxed from equality to inequality and overtake a role of a physical admissibility condition for weak solutions. We want to discuss what is a critical regularity of weak solutions to a general system of conservation laws to satisfy an associated companion law as an equality. An archetypal example of such result was derived for the incompressible Euler system by Constantin et al. ([1]) in the context of the seminal Onsager's conjecture. This general result can serve as a simple criterion to numerous systems of mathematical physics to prescribe the regularity of solutions needed for an appropriate companion law to be satisfied.

References

[1] P. Constantin, W. E, and E. S. Titi. Onsager's conjecture on the energy conservation for solutions of Euler's equation. *Comm. Math. Phys.*, 165(1):207–209, 1994.

[2] Feireisl, Eduard; Gwiazda, Piotr; Świerczewska-Gwiazda, Agnieszka; Wiedemann, Emil; Regularity and Energy Conservation for the Compressible Euler Equations, *Arch. Ration. Mech. Anal.* 223 (2017), no. 3, 1375–1395.

[3] P. Gwiazda, M. Michálek, A. Świerczewska-Gwiazda. A note on weak solutions of conservation laws and energy/entropy conservation, arXiv:1706.10154.

[4]T. Debiec, P. Gwiazda, A. Świerczewska-Gwiazda; A tribute to conservation of energy for weak solutions, arXiv:1709.01410

On random distributions and applications to compressible fluids

Martina Hofmanová, D. Breit, E. Feireisl

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Abstract

In this talk, we review the basic issues of stochastic integration in connection with the Navier-Stokes system for compressible fluids. In particular we discuss the theory of Ito stochastic integration in the context of random distributions.

On non-uniqueness of the two dimensional compressible Euler equations

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Abstract

In this lecture will shall address the question of uniqueness of solutions to the two dimensional compressible Euler equations. The aim is to investigate if there exists a unique entropy solution or if the convex integration method produces infinitely many entropy solutions. Using this method we shall depict non-uniqueness for both the isentropic and the full Euler equations. We shall show that this can be done for various initial data.

Self-similar solutions to a thin film Muskat problem

Philippe Laurençot

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Abstract

The thin film Muskat describes the space-time evolution of the heights of two layers of non-miscible fluids with different viscosity and density and is a second-order parabolic system featuring a diffusion matrix which is full and degenerate. A complete classification of self-similar solutions is provided and their role in the large time behaviour is investigated. Joint work with Bogdan-Vasile Matioc (Hannover).

Visualization of the convex integration solutions to the Monge-Ampère equation

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Abstract

We implement the algorithm based on the convex integration result and obtain visualizations of the first iterations of the Nash-Kuiper scheme, approximating the anomalous solutions to the Monge-Ampere equation in two dimensions.

The role of measure-valued solutions in compressible fluids.

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Abstract

In the present talk we will concentrate on the question of convergence of suitable numerical schemes for both viscous and inviscid compressible flows. A standard paradigm for the existence of solutions in fluid dynamics is based on the construction of sequences of approximate solutions or numerical schemes. However, if the underlying model does not provide enough information for the required regularity of the approximate sequence, we are facing the problem to show the scheme's convergence. In particular, for multidimensional problems fine scale oscillations persist, which prevents us to obtain compactness result. Consequently, the standard framework of integrable functions seems not be appropriate in general.

To overcome this problem we introduce the class of dissipative measure-valued solutions, which allows us to show the convergence of finite volume or combined finite volume-finite element schemes for multidimensional isentropic Euler and Navier-Stokes equations, respectively. On the other hand, using the weak-strong uniqueness result for the above systems we know, that the dissipative measure-valued solution coincides with the strong solution if the latter exists. Consequently, our results show convergence of our numerical schemes to the strong solutions.

On Euler/Navier-Stokes fluids

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Abstract

The Euler and Navier-Stokes equations are intesively studied by mathematical analysts for decades. Despite this fact, there is a class of fluids that we call Euler/Navier-Stokes fluids that seems completely overlooked in the history of fluid mechanics. These fluids are characterized by the following dichotomy: (i) when the shear rate is below a certain critical value the fluid behaves as the Euler fluid (i.e. there is no effect of the viscosity, the shear stress vanishes), on the other hand (ii) if the shear rate exceeds the critical value, dissipation takes place and fluid can respond as shear (or stress) thinning or thickening fluid or as a Navier-Stokes fluid. Implicit constitutive theory provides an elegant framework to express such responses involving the activation criterion in a compact and elegant manner.

In the lecture, we present mathematical theory available for these class of fluids flowing in bounded smooth domains. We subject such flows to different types of boundary conditions including no-slip, Navier's slip and activated boundary conditions like stick-slip.

The lecture is based on a joint work with Jan Blechta and K.R. Rajagopal on the classification of fluids, part 1: incompressible fluids.

Splash singularity for a free-boundary incompressible viscoelastic fluid model

Pierangelo Marcati, E. Di Iorio and S. Spirito

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Abstract

We analyze a 2D free-boundary viscoelastic fluid model of Oldroyd- B type at infinite Weissenberg number. Our main goal is to show the existence of the so-called splash singularity, namely a point where the boundary remains smooth but self-intersects.

The investigation starts with the regularization of the initial splash domain by mapping this domain via a conformal transformation in order to obtain a smooth initial conguration and then by using a lagrangian change of variables to fix the boundary. We prove the local existence and stability results for this smooth initial conguration and, by constructing a special class of initial data, we show that it evolves into a self-intersecting configuration. These results include previous ones obtained by A. Castro, D. Cordoba, C. Feerman, F. Gancedo & J.Gomez-Serrano and by D.Coutand and S.Shkoller, for the Navier Stokes equation.

As a consequence we can conclude that there exists a time where the conguration forms a splash type singularity. In conclusion we show further extension of these results to more general Piola type stress tensors and to large but finite Weissenberg numbers.

Existence of weak solutions to compressible Navier-Stokes equations with large non-homogenous data

Antonín Novotný

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Abstract

We shall discuss the weak solvability of compressible Navier-Stokes equations with large non-homogenous boundary data in the case of barotropic and hard sphere constitutive laws for pressure.

A rigorous derivation of the stationary compressible Reynolds equation via the Navier-Stokes

Adrien Petrov, I. S. Ciuperca, E. Feireisl, M. Jai

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Abstract

The talk deals with a rigorous derivation of the compressible Reynolds system as a singular limit of the compressible (barotropic) Navier-Stokes system on a thin domain. In particular, the existence of solutions to the Navier-Stokes system with non-homogeneous boundary conditions is presented. The approach is based on new a priori bounds available for the pressure law of hard sphere type. The uniqueness result for the limit problem in the one-dimensional case is also discussed

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Compressible Navier-Stokes equations with entropy transport

Milan Pokorný, D. Maltese, M. Michálek, P. Mucha, A. Novotný, E. Zatorska

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Abstract

We consider the compressible Navier–Stokes system with variable entropy. The pressure is a nonlinear function of the density and the entropy/potential temperature which, unlike in the Navier–Stokes–Fourier system, satisfies only the transport equation. We provide existence results within three alternative weak formulations of the corresponding classical problem. Our constructions hold for the optimal range of the adiabatic coefficients from the point of view of the nowadays existence theory.

Regularity and uniqueness for a critical Ladyzhenskaya fluid

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Abstract

We consider an incompressible p-law type fluid in a 3D bounded domain. Employing iterative estimate in Nikolskii spaces and reverse Hoelder inequality, we establish higher time regularity and uniqueness of weak solution provided the data are more regular.

Entire solutions of a semilinear parabolic equation

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Abstract

Entire solutions (defined for all positive and negative times) play an important role in the study of singularities and long-time behavior of solutions of many evolution problems. We are mainly interested in entire, positive, radially symmetric solutions of a nonlinear heat equation with a power nonlinearity. The existence and properties of such solutions strongly depend on the exponent of the nonlinearity. We will discuss the corresponding (well-known and new) results and provide some of their applications.

Diffuse interface models for multiphase tumor growth

Elisabetta Rocca, S. Frigeri, K.-F. Lam, G. Schimperna

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Abstract

In this talk we report on a recent joint work [S. Frigeri, K.-F. Lam, E. Rocca, G. Schimperna, On a multi-species Cahn-Hilliard-Darcy tumor growth model with singular potentials, preprint arXiv:1709.01469 (2017)]. We consider a model describing the evolution of a tumor inside a host tissue in terms

of the proliferating and dead cells, the cell velocity and the nutrient concentration. The tumor phase variables satisfy a Cahn–Hilliard type system with nonzero forcing term (implying that their spatial means are not conserved in time), whereas the velocity obeys a form of the Darcy law and the nutrient parameter satisfies a quasi-static diffusion equation. The main novelty of the present work stands in the fact that we are able to consider a configuration potential of singular type implying that the concentration vector is constrained to remain in the range of physically admissible values.

On the other hand, in view of the presence of nonzero forcing terms, this choice gives rise to a number of mathematical difficulties, especially related to the control of the mean values of the tumor phases. For the resulting mathematical problem, by imposing suitable initial-boundary conditions, our main result concerns

the existence of weak solutions in a proper regularity class.

Some results on the functionalized Cahn-Hilliard equation

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Abstract

We will consider the so-called "functionalized Cahn-Hilliard equation" settled on a bounded subset of the Euclidean space. This is a sixth-order variant of the standard Cahn-Hilliard model which describes phase-separation processes in some classes of polymeric materials. We will discuss the case when the configuration potential F for the phase variable has a singular character, namely its derivative F' explodes logarithmically fast in proximity of the pure phase configurations. This situation is mathematically difficult in view of the fact that, even for convex F, the operator $F'(\cdot) + \Delta^2$ appearing in the Cahn-Hilliard equation is not maximal monotone in L^2. We will show how this difficulty can be overcome and discuss various mathematical results related to suitable weak formulations of the problem, existence of solutions, regularity, and uniqueness.

Global weak solution to 1D compressible MHD without resistivity

Yongzhong Sun

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Abstract

In this talk I will give some recent results on one dimensional compressible MHD system without resistivity, mainly concerns with the existence and uniqueness of global weak solution and its long time behavior, without size restriction on the initial data. These results are obtained under Lagrangian formulation, which is the same as the classical works for 1D compressible Navier-Stokes equations.

Transport of congestion in two phase compressible/incompressible flow

Ewelina Zatorska, D. Bresch, C. Perrin, P. Degond, P. Minkowski, and L. Navoret

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Abstract

Can the fluid equations be used to model pedestrian motion or traffic?

In this talk, I will present the compressible-incompressible two phase system describing the flow in the free and in the congested regimes. I will show how to approximate such system by the compressible Navier-Stokes equations with singular pressure for the fixed barrier densities, together with some recent developments for the barrier densities varying in the space and time.

At the end, I will present the numerical results showing that our macroscopic system captures some features characteristic for microscopic models of collective behaviour.

Dissipative measure valued solutions for general hyperbolic conservation laws

Agnieszka Świerczewska-Gwiazda, P. Gwiazda, O. Kreml

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Abstract

In the last years measure-valued solutions started to be considered as a relevant notion of solutions if they satisfy the so-called measure-valued – strong uniqueness principle. This means that they coincide with a strong solution emanating from the same initial data if this strong solution exists. Following result of Yann Brenier, Camillo De Lellis and Laszlo Szekelyhidi Jr. for incompresible Euler Equation, this property has been examined for many systems of mathematical physics, including incompressible and compressible Euler system, compressible Navier-Stokes system, polyconvex elastodynamics et al. One observes also some results concerning general hyperbolic systems. Our goal is to provide a unified framework for general systems, that would cover the most interesting cases of systems. Additionaly following the result [3] for compressible Navier-Stokes system we introduce a concept of dissipative measure valued solution to general hyperbolic systems.

References

[1] Brenier, Y., De Lellis, C., SzÈkelyhidi Jr., L., Weak-strong uniqueness for measure-valued solutions. Comm. Math. Phys. 305(2), 351–361 (2011)

[2] S. Demoulini, D. M. A Stuart, and A. Tzavaras, Weak-strong uniqueness of dissipative measure-valued solutions for polyconvex elastodynamics, Arch. Ration. Mech. Anal. 205 (2012), no. 3, 927–961

[3] E. Feireisl, P. Gwiazda, A. Świerczewska-Gwiazda and Emil Wiedemann Dissipative measure-valued solutions to the compressible Navier-Stokes system, Calc. Var. Partial Differential Equations, 55 (2016), no. 6, 55–141

[4] P. Gwiazda, A. Świerczewska-Gwiazda and E. Wiedemann, Weak-Strong Uniqueness for Measure-Valued Solutions of Some Compressible Fluid Models, Nonlinearity 28 (2015), no. 11, 3873–3890.

[5] T. Debiec, P. Gwiazda, K. Łyczek, A. Świerczewska-Gwiazda, A tribute to conservation of energy for weak solutions, to appear in Topol. Methods Nonlinear Anal.

Convergence to travelling waves in Fisher's population genetics model with degenerate diffusion and a non-Lipschitzian reaction term

Peter Takáč, P. Drábek

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Abstract

We begin by a brief presentation of a well-known mathematical model for population genetics due to R. A. Fisher (1937) where a population is divided into three classes of genotypes, aa, AA, and aA. The mathematical model is represented by a reaction-diffusion equation for the unknown *relative density* $u(x,t)$ of the population of allele A at the point $x \in \mathbb{R}$ of the habitat at time $t \in \mathbb{R}_+$. An important question from Population Biology is if *genetic diversity* is preserved at certain location and at certain time. We will show that this is a "dynamic problem" that requires a dynamical system approach by convergence to a travelling wave.

Our first mathematical result will be on travelling waves with a degenerate or singular diffusion (like the p-Laplacian) and possibly nonsmooth (e.g., non-Lipschitzian), bi-stable reaction term. We will show that, in spite of nonuniqueness for the one-dimensional ordinary differential equation, the travelling wave $u(x,t) = v(x - ct)$ and its speed c are unique (up to a spatial shift in the argument). In order to answer our genetic diversity problem, we have to know if the two extreme values 0 and 1 of the solution (i.e., the profile of travelling wave) $v(x)$ are reached within a bounded spatial interval $[z_0, z_1] \subset \mathbb{R}$. Namely, the genetic diversity is lost in $(-\infty, z_0]$ thanks to $v = 0$, and in $[z_1, \infty)$ thanks to $v = 1$.

Our second result establishes the long-time convergence to a travelling wave for the classical Brownian diffusion (i.e., the linear Laplace operator) combined with a non-Lipschitzian reaction term. We will briefly explain the "mechanism" that yields the desired convergence.

Localised Relative Energy and Finite Speed of Propagation

Emil Wiedemann

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Abstract

The relative entropy method, introduced by C. Dafermos in the context of hyperbolic conservation laws, has been successfully applied to the study of uniqueness problems and singular limits in compressible fluid dynamics, among others. We will present a space-localised version of the relative entropy (or rather energy) method for the compressible Euler equations, and show how it can be applied to questions concerning finite speed of propagation for weak solutions.

The list of invited speakers:

- Peter Bella, Max Planck Institute, Leipzig, Peter.Bella@math.uni-leipzig.de
- Jan Březina, Tokyo Institute of Technology, h.brezina@gmail.com
- Dorin Bucur, University of Savoie, dorin.bucur@univ-savoie.fr
- Elisabetta Chiodaroli, University of Pisa, elisabetta.chiodaroli@epfl.ch
- Donatella Donatelli, University of L'Aquila, donatell@univaq.it
- Piotr Gwiazda, IMPAN, Warsaw, pgwiazda@mimuw.edu.pl
- Martina Hofmanová, Technical University Berlin, hofmanov@math.tu-berlin.de
- Christian Klingenberg, University of Würzburg, klingen@mathematik.uni-wuerzburg.de
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