

# A $p(x)$ -Laplacian Extension of a Convexity Result and Applications to Quasi-linear Elliptic BVPs

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## Abstract

The main result of this work is a new extension of the well-known inequality by Díaz and Saa which, in our case, involves an anisotropic operator, such as the  $p(x)$ -Laplacian,  $\Delta_{p(x)}u \equiv \operatorname{div}(|\nabla u|^{p(x)-2}\nabla u)$ . Our present extension of this inequality enables us to establish several new results on the uniqueness of solutions and comparison principles for some anisotropic quasilinear elliptic equations. Our proofs take advantage of certain convexity properties of the energy functional associated with the  $p(x)$ -Laplacian.

**Running head:** Díaz and Saa Inequality for  $\Delta_{p(x)}$

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quasilinear Dirichlet problem with variable exponents;  
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uniqueness and comparison principles

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