Kernel-based approximation methods for data analysis and machine learning

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Abstract: Kernel-based approximation methods are powerful tools for the analysis of scattered data f_X that are sampled from a scalar-valued multivariate function f at finitely many evaluation functionals X. So do kernel-based approximation schemes in particular provide meshfree discretizations for the numerical solution of partial differential equations, e.g. particle-based advection methods or collocation schemes.

In this talk, we report on recent progress concerning kernel-based learning methods, which could be viewed as one particular variant of linear regression. Kernel-based learning is particularly relevant, if the input data (X, f_X) are very large and decontaminated from noise. In such application scenarios, we wish to reduce, for a suitable function space $\mathcal{R}$, the empiric ℓ^2-data error

$$\eta_X(f,s) = \frac{1}{|X|}\|s_X - f_X\|_2^2$$

under variation of $s \in \mathcal{R}$. To this end, we construct an approximation $s^* \in \mathcal{R}$ to f, $s^* \approx f$, which satisfies specific smoothness requirements, where the smoothness is measured by an appropriate functional $J : \mathcal{R} \longrightarrow [0, \infty)$.

To make a compromise between the empiric data error $\eta_X(f,s)$ and the smoothness $J(s)$, for $s \in \mathcal{R}$, we consider minimizing the cost functional

$$J_\alpha(s) = \eta_X(f,s) + \alpha J(s) \qquad \text{for } s \in \mathcal{R},$$

where $\alpha > 0$ is used to balance between the data error $\eta_X(f,s)$ and the smoothness $J(s)$ of s. Note that this regression method can also be viewed as a regularization method, or, in the jargon of approximation theory, as *penalized least squares approximation* (PLSA). This talk addresses important aspects concerning the stability and the convergence of kernel-based PLSA.