Testing Parametric Mean Specifications in Semiparametric GARCH-in-Mean Models.

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Abstract

We consider time series models in which the conditional mean of a response variable Y_t given the past $\mathcal{F}_t$ depends on an unobserved covariate h_t. More precisely, we assume that for $t = 1, ..., T$:

$$Y_t = m_0(h_t) + \varepsilon_t,$$

where ε_t fulfills $\mathbf{E}[\varepsilon_t|\mathcal{F}_{t-1}] = 0$ for an increasing σ-field $\mathcal{F}_t$ with the property that (ε_t, h_{t+1}) is $\mathcal{F}_t$-measurable.

The covariate process h_t is an unobserved one-dimensional process. We assume that h_t can be consistently estimated by known functions $\widehat{h}_t$ that depend on parameters ψ and m and on the past observations $Y_{t-1}, Y_{t-2}, \ldots$. We denote the true parameter values by ψ_0 and m_0, i.e. $h_t = \widehat{h}_t(\psi_0, m_0)$. A typical example could be that h_t follows a GARCH$(1, 1)$ process or another specification from the GARCH family. Most importantly, we allow h_t to depend on the function m_0. In particular, this is the case if $\widehat{h}_t$ depends on ψ_0 and on the residuals $\varepsilon_1, ..., \varepsilon_{t-1}$. Then one needs the function m_0 to get the residuals from the observations $Y_{t-1}, Y_{t-2}, \ldots$. Our central assumption on $\widehat{h}_t$ is that this function is measurable with respect to $\mathcal{F}_{t-1}$.

In our talk we discuss testing parametric specifications of m. Our testing procedure is based on iterative fits of the covariate and nonparametric kernel smoothing of the conditional mean function. The test statistic is given by the L_2 norm of the difference between the kernel estimator and the parametric estimator of m. We show that this test statistic is asymptotically normal and discuss its asymptotic power. We apply the theory to tests for parametric GARCH-M models. For this purpose we have to develop asymptotic theory for this model that is not available up to now. The proofs of our results are based on empirical process methods.

We apply our approach for testing economic theories that postulate functional relations between macroeconomic or financial variables and their conditional second moments. We illustrate the usefulness of the methodology by testing the linear risk-return relation predicted by the ICAPM.

The talk reports on joint work with Christian Conrad, Heidelberg.

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