On a stochastic $p(\omega,t,x)$-Laplace equation

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In this talk we present recent results (obtained in joint work with Guy Vallet, Pau, France, and Aleksandra Zimmermann, Essen) on existence and uniqueness of solutions to the problem $\mathrm{P}(u_0,h)$:

$$
\begin{aligned}
du - \Delta_{p(\cdot)} u \, dt = h(\cdot,u) \, dW \qquad & \text{in } \Omega \times (0,T) \times D, \\
u = 0 \qquad & \text{on } \Omega \times (0,T) \times \partial D, \\
u(0,\cdot) = u_0 \qquad & \text{in } D,
\end{aligned}
$$

where $T > 0$, $D \subset R^d$ is a bounded Lipschitz domain, $Q := (0,T) \times D$, $W = \{W_t, \mathscr{F}_t; 0 \leq t \leq T\}$ is a Wiener process on a probability space $(\Omega, \mathscr{F}, P)$, $h = h(\omega,t,x,\lambda)$ is a Carathéodory function on $\Omega \times Q \times R$, uniformly Lipschitz continuous with respect to λ, $\Delta_{p(\cdot)} u = \mathrm{div}(|\nabla u|^{p(\cdot)-2} \nabla u)$ is the p-Laplace operator with a variable exponent $p : \Omega \times Q \to (1,\infty)$ satisfying some further appropriate conditions.

Problems with variable exponent (i.e., when the Lebesgue exponent p depends on the time-space arguments) have been intensively studied since around the beginning of this century. The main physical motivation was induced by the modelisation of electrorheological fluids. In this case one studies the case of a coupled problem, where in one equation for the unknown velocity field u the exponent $p = p(v(t,x))$ depends on a solution v of a coupled PDE. Since reality is complex and measurements of given data are rarely exact, it can be interesting to consider models with stochastic perturbations acting on both equations, i.e.,

$$du + A(u,v) \, dt = f \, dW, \quad dv + B(v) \, dt = g \, dW.$$

This is one motivation for our interest to study the toy problem $\mathrm{P}(u_0,h)$ with variable exponent p depending also on ω which arises when $A(u,v) = -\Delta_{p(v(\omega,t,x))}(u)$.