Sharp a priori estimates and uniqueness results for a class of nonlinear elliptic equations

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ABSTRACT. I present sharp a prori estimates and uniqueness results for second order quasi-linear elliptic operators in divergence form with a first order term. Precisely, let Ω be a bounded open set of $\mathbb{R}^N$, $N \geq 2$. We consider the model problem

$$\begin{cases} -\Delta_p u = G(x, \nabla u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{0.1}$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ denotes the p-laplacian operator, $1 < p < N$, and $G(x, \xi)$ is a function which, for $p - 1 < q < p$, satisfies the growth condition

$$|G(x, \xi)| \leq \beta|\xi|^q + f(x) \tag{0.2}$$

with β a positive constant and f a summable function.

Our aim is to study problems in the form (0.1) when the exponent q in (0.2) varies in the given interval and we are mainly interested to find sharp conditions on f which guarantee the existence of a solution. Also uniqueness results are presented.