

On the Heston model in mathematical finance: An analytic approach by PDEs.

We study the Heston model for pricing European options on stocks with stochastic volatility. This is a Black-Scholes-type equation whose spatial domain is the half-plane $\mathbb{H} = \mathbb{R} \times (0, \infty)$; the equation is parabolic and degenerates at the boundary $\partial\mathbb{H} = \mathbb{R} \times \{0\}$. However, we are able to show that the Black-Scholes-type operator, which appears in this equation, generates a holomorphic C^0 -semigroup in a suitable weighted L^2 -space over \mathbb{H} . We show that the C^0 -semigroup solution can be extended to a holomorphic function in a complex domain.