Sign of the solution to a non-cooperative system

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(joint work with Bénédicte Alziary)

Many results have been obtained since decades on Maximum Principle and Antimaximum principle for second order elliptic partial differential equations involving *e.g.* Laplacian, p-Laplacian, Schrödinger operator, ... or weighted equations. Then most of these results have been extended to systems.
The maximum principle (studied since centuries) has many applications in various domains as physic, chemistry, biology,...Usually it shows that for positive data the solutions are positive (positivity is preserved). It is generally valid for a parameter below the "principal" eigenvalue (the smallest one).
The Antimaximum principle, introduced in 1979 by Clément and Peletier , shows that, for one equation, as this parameter goes through this principal eigenvalue, the sign are reversed; this holds only for a small interval.

Here we recall and then extend these results to $n \times n$ systems involving Dirichlet Laplacian. These systems are not necessarily cooperative (all the terms outside the diagonal of the associated square matrix are not necessarily positive).

First we recall precise results concerning one equation: Let Ω be a smooth bounded domain in $I\!R^N$. Consider the following Dirichlet boundary value problem

$$-\Delta z \; = \; \sigma z + \; h \; \text{ in } \Omega \; , \;\; z = 0 \text{ on } \partial\Omega, \tag{0.1}$$

where σ is a real parameter. We study the sign of z when $h \geq 0$ and σ near the first eigenvalue of the Dirichlet Laplacian.

Then we consider a $n \times n$ (eventually non-cooperative) system defined on Ω a smooth bounded domain in $I\!R^N$:

$$-\Delta U = AU + \mu U + F \text{ in } \Omega \, , \; U = 0 \text{ on } \partial\Omega, \tag{S}$$

where F is a column vector with components $f_i \geq 0$, $1 \leq i \leq n$. We obtain several results on the sign of the components of U when μ varies in some interval.